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Angular Distribution and Polarization of Photons in the Inclusive Decay $\Lambda_b \to X_s\,\gamma$

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ABSTRACT

We study the angular distribution of photons produced in the inclusive decay of a polarized Λ_b, $\Lambda_b \to X_s\,\gamma$, using the technique of heavy quark effective theory. Finite non-perturbative corrections are obtained relative to the free quark decay $b \to s\gamma$. These corrections affect significantly the intensity and polarization of photons emitted at small angles relative to the Λ_b spin direction.

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I. INTRODUCTION

The decay $b \to s\gamma$, calculated on the level of free quarks, has three interesting features:

(i) The photon is monochromatic, its energy spectrum being

$$\left(\frac{1}{\Gamma}\frac{d\Gamma}{dy}\right)_{\rm FQM} = \delta(y-y_0) \ , \quad y = \frac{2E_\gamma}{m_b} \ , \quad y_0 = \left(1-\frac{m_s^2}{m_b^2}\right) \ . \tag{1.1}$$

(ii) The photon is emitted preferentially backwards relative to the spin of the b-quark, the angular distribution being

$$\left(\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}\right)_{\rm FQM} = \frac{1}{2}\left(1-\frac{m_b^2-m_s^2}{m_b^2+m_s^2}\cos\theta\right) \ . \tag{1.2}$$

(iii) The photon is predominantly left-handed, the angular distribution for helicities $\lambda_\gamma = \pm 1$ being

$$\begin{aligned} \left(\frac{1}{\Gamma}\frac{d\Gamma_+}{d\cos\theta}\right)_{\rm FQM} &= \frac{1}{2}\frac{m_s^2}{m_b^2}\,(1+\cos\theta)\left(1+\frac{m_s^2}{m_b^2}\right)^{-1} \ , \\ \left(\frac{1}{\Gamma}\frac{d\Gamma_-}{d\cos\theta}\right)_{\rm FQM} &= \frac{1}{2}\,(1-\cos\theta)\left(1+\frac{m_s^2}{m_b^2}\right)^{-1} \ , \\ P \equiv \frac{\Gamma_+-\Gamma_-}{\Gamma_++\Gamma_-} &= -\frac{m_b^2-m_s^2}{m_b^2+m_s^2} \ . \end{aligned} \tag{1.3}$$

Whereas feature (i) is purely kinematical (reflecting the two-body nature of the decay), features (ii) and (iii) are specific consequences of the standard model, in which the effective Hamiltonian governing $b \to s\gamma$ has the structure

$$\begin{aligned} H_{\rm eff} &= \frac{-4G_F}{\sqrt{2}}\frac{e}{16\pi^2}\,V_{tb}V_{ts}^*\,c_7(m_b)\,\bar{s}\,\sigma^{\mu\nu}\,(m_bP_R+m_sP_L)\,bF_{\mu\nu} \\ &\equiv \frac{-4G_F}{\sqrt{2}}\frac{e}{16\pi^2}\,V_{tb}V_{ts}^*\,c_7(m_b)\,\bar{s}\,\Gamma^{\mu\nu}\,bF_{\mu\nu} \ , \end{aligned} \tag{1.4}$$

leading to a decay width

$$\Gamma_{\rm FQM}(b\to s\gamma) = \frac{\alpha G_F^2 m_b^5}{32\pi^4}\,|V_{tb}V_{ts}^*|^2\,|c_7(m_b)|^2\,(1+\frac{m_s^2}{m_b^2})(1-\frac{m_s^2}{m_b^2})^3 \ . \tag{1.5}$$

The purpose of this paper is to study how the characteristics of $b \to s\gamma$, summarized in Eqs. (1.1)–(1.3), are altered when the b-quark is embedded in a polarized Λ_b baryon, and when the final s-quark is a part of a hadronic system that is summed over. These changes have their origin in the "Fermi-motion" of the b-quark within the hadron, as well as its spin-dependent interaction with the environment, and can be parametrized using the method of heavy quark effective theory (HQET) [1,2]. In Sec. II, we outline the method of calculation, and discuss our results in Sec. III.

II. HQET AND THE DECAY $\Lambda_b \to X_s\,\gamma$

The differential decay rate for the inclusive decay $\Lambda_b(p) \to X_s(p_X)\,\gamma(p_\gamma)$ in the standard model can be written as

$$\begin{aligned} d\Gamma &= \sum_{X_s,\,\mathrm{pol}} (2\pi)^4 \delta^4(p - p_X - p_\gamma)\, \frac{d^3\mathbf{p}_X}{(2\pi)^3 2E_X} \\ &\quad \times \langle \Lambda_b |\, H^\dagger_{\mathrm{eff}}(0)\, | X_s\gamma \rangle \, \langle X_s\gamma |\, H_{\mathrm{eff}}(0)\, | \Lambda_b \rangle \, \frac{d^3\mathbf{p}_\gamma}{(2\pi)^3 2E_\gamma}\ , \end{aligned} \tag{2.1}$$

where the effective Hamiltonian H_{eff} is given in (1.4). Using the optical theorem, and writing the phase space element as

$$\frac{d^3\mathbf{p}_\gamma}{(2\pi)^3 2E_\gamma} = \frac{m_b^2}{32\pi^2}\, y\, dy\, d\cos\theta\ , \tag{2.2}$$

Eq. (2.1) can be rewritten in the form

$$\frac{d\Gamma}{dy\, d\cos\theta} = \frac{\alpha G_F^2 m_b^2}{2^7 \pi^5}\, |V_{tb} V_{ts}^*|^2\, |c_7(m_b)|^2\, y\, \mathrm{Im}\, T(y, \cos\theta)\ , \tag{2.3}$$

where T is given by

$$T = E_{\mu\nu\alpha\beta} T^{\mu\nu\alpha\beta}\ ,$$

with

$$E_{\mu\nu\alpha\beta} \equiv \sum_{\lambda} \langle 0| F_{\mu\nu} |\gamma\rangle \langle\gamma| F_{\alpha\beta} |0\rangle \ , \tag{2.4}$$

and

$$T_{\mu\nu\alpha\beta} = i \int d^4x \ e^{-ip_\gamma \cdot x} \langle \Lambda_b | \, \mathrm{T} \left\{ \bar{b}(x) \Gamma^{\dagger}_{\mu\nu} s(x), \bar{s}(0) \Gamma_{\alpha\beta} b(0) \right\} |\Lambda_b\rangle \ . \tag{2.5}$$

The time-ordered product appearing in $T_{\mu\nu\alpha\beta}$ can be expanded in powers of $1/m_b$, using methods described in [2–8]. To order $1/m_b^2$, we obtain

$$\begin{aligned} T(y, \cos\theta) &= 2y^2 m_b^3 \, \frac{1}{(y - y_0 - i\epsilon)} \\ \times & \left\{ \left[1 + K \left(\frac{5}{3} - \frac{7}{3} \frac{y}{(y - y_0 - i\epsilon)} + \frac{2}{3} \frac{y^2}{(y - y_0 - i\epsilon)^2} \right) \right] \left(1 + \frac{m_s^2}{m_b^2} \right) \right. \\ - & \left. \cos\theta \left[1 + \epsilon_b + K \left(\frac{5}{3} - \frac{7}{3} \frac{y}{(y - y_0 - i\epsilon)} + \frac{2}{3} \frac{y^2}{(y - y_0 - i\epsilon)^2} \right) \right] \left(1 - \frac{m_s^2}{m_b^2} \right) \right\} \ . \end{aligned} \tag{2.6}$$

The imaginary part Im $T(y, \cos\theta)$ is then obtained by the formal replacement

$$\begin{aligned} \frac{1}{y - y_0 - i\epsilon} \quad &\rightarrow \quad \pi \, \delta(y - y_0) \ , \\ \frac{1}{(y - y_0 - i\epsilon)^2} \quad &\rightarrow \quad -\pi \, \delta'(y - y_0) \ , \\ \frac{1}{(y - y_0 - i\epsilon)^3} \quad &\rightarrow \quad \frac{\pi}{2} \, \delta''(y - y_0) \ . \end{aligned} \tag{2.7}$$

The distributions in Eq. (2.7) yield physically meaningful results only if one integrates over the photon energy (see e.g. Ref. [8]). The leading term proportional to $\delta(y - y_0)$ reproduces the result of the free quark model (FQM). The corrections to the free quark result, involving $\delta'(y - y_0)$ and $\delta''(y - y_0)$, are parametrized by two phenomenological constants K and ϵ_b, defined as

$$K \ = \ - \langle \Lambda_b | \, \bar{h}_v \frac{(iD)^2}{2m_b^2} h_v \, |\Lambda_b\rangle \ , \tag{2.8}$$

$$(1 + \epsilon_b) s^\mu \ = \ \langle \Lambda_b | \, \bar{b} \, \gamma^\mu \gamma^5 b \, |\Lambda_b\rangle \ . \tag{2.9}$$

(Refs. [4, 8] use a parameter λ_1 related to K by $K = -\lambda_1/2m_b^2$). Integration of Eq. (2.3) over y then yields the angular distribution

$$\frac{d\Gamma(\Lambda_b \to X_s\,\gamma)}{d\cos\theta} = \frac{1}{2}\Gamma_{\rm FQM}\left[(1-K) - (1+\epsilon_b - K)\,\frac{m_b^2 - m_s^2}{m_b^2 + m_s^2}\,\cos\theta\right]\ . \qquad (2.10)$$

If the decay $\Lambda_b \to X_s\,\gamma$ is calculated for a fixed photon helicity ($\lambda_\gamma = \pm 1$), the angular distributions are

$$\frac{d\Gamma_+(\Lambda_b \to X_s\,\gamma)}{d\cos\theta} = \frac{1}{2}\Gamma_{\rm FQM}\left(\frac{m_s^2}{m_b^2}\right)[(1-K) + (1+\epsilon_b - K)\cos\theta]\left(1+\frac{m_s^2}{m_b^2}\right)^{-1}\ , \qquad (2.11)$$

$$\frac{d\Gamma_-(\Lambda_b \to X_s\,\gamma)}{d\cos\theta} = \frac{1}{2}\Gamma_{\rm FQM}\,[(1-K) - (1+\epsilon_b - K)\cos\theta]\left(1+\frac{m_s^2}{m_b^2}\right)^{-1}\ . \qquad (2.12)$$

Eqs. (2.10)–(2.12) are the HQET analogs of the free quark result (1.2)–(1.3).

III. DISCUSSION OF RESULTS

To evaluate the HQET corrections, we need an estimate of the parameters K and ϵ_b. Following Ref. [5] we use $K \simeq 0.01$, although this parameter has a large uncertainty. From the fact that $d\Gamma/d\cos\theta \geq 0$ for all scattering angles (independent of the value of m_s or K) we must have $\epsilon_b < 0$ (see Eq. (2.10)). We will use the value $\epsilon_b = -\frac{2}{3}K$ suggested in Ref. [3].[1] This is in agreement with the bound derived in Ref. [9].

From the angular distribution in Eq. (2.10), we infer that the effect of non-perturbative corrections is to reduce slightly the forward-backward asymmetry in the photon emission. In Fig. 1, we plot the fraction of decays producing a photon

[1]This value is obtained if one neglects the contributions coming from the double insertion of the chromomagnetic operator [3].

in the forward cone $\cos\theta_0 < \cos\theta < 1$:

$$I(\cos\theta_0) \equiv \frac{\int\limits_{\cos\theta_0}^{1} d\cos\theta \, \frac{d\Gamma}{d\cos\theta}}{\int\limits_{-1}^{+1} d\cos\theta \, \frac{d\Gamma}{d\cos\theta}} \ . \tag{3.1}$$

The result is compared with that in the free quark model.

From the results in Eqs. (2.11) and (2.12), we obtain the polarization of the photon as a function of its direction:

$$P(\cos\theta) = \frac{d\Gamma_+/d\cos\theta - d\Gamma_-/d\cos\theta}{d\Gamma_+/d\cos\theta + d\Gamma_-/d\cos\theta} \ . \tag{3.2}$$

Once again, as shown in Fig. 2, corrections to the free quark model are significant for photons emitted in the near-forward direction.

To summarize, the angular distribution of inclusively produced photons in the decay $\Lambda_b \to X_s\,\gamma$ of a polarized Λ_b baryon can be calculated unambiguously in HQET. This distribution tests the structure of the underlying Hamiltonian $H_{\rm eff}$ in a way that is not possible by studying the mesonic decay $B \to X_s\,\gamma$. The angular distribution is a well defined observable which involves no divergences of the type that appear in the calculation of the energy spectrum (delta functions and derivatives thereof). Our calculation may be viewed as an illustration of the utility (and limitation) of the HQET approach in determining the effects of hadronic binding on the decay of a heavy quark. The deviations from free quark decay in the reaction $\Lambda_b \to X_s\,\gamma$ are found to be globally small, but are significant for photons emitted in the forward direction.

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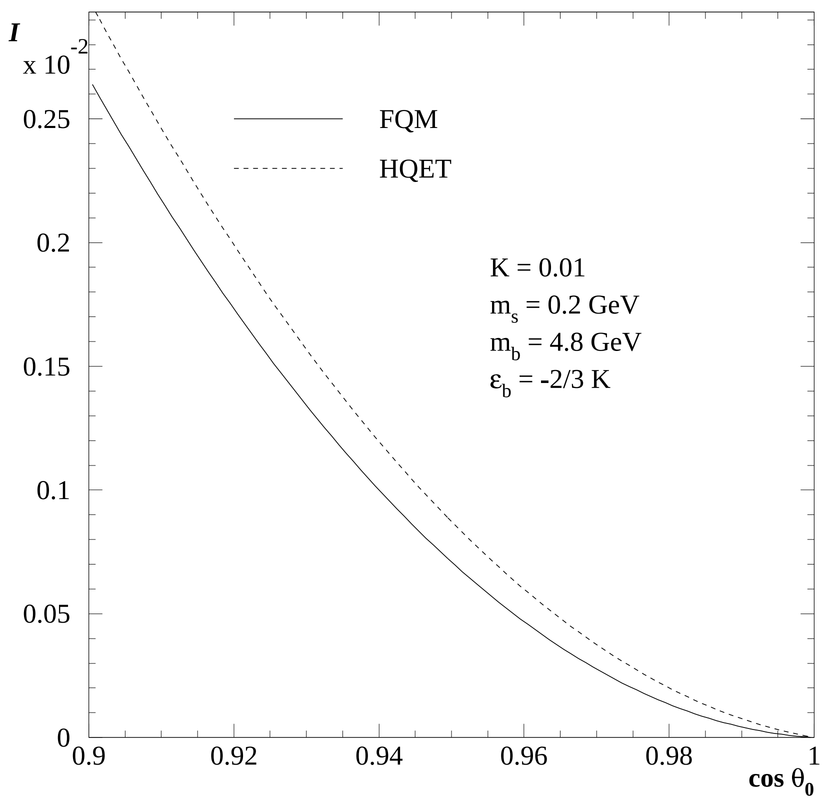
FIGURE CAPTIONS

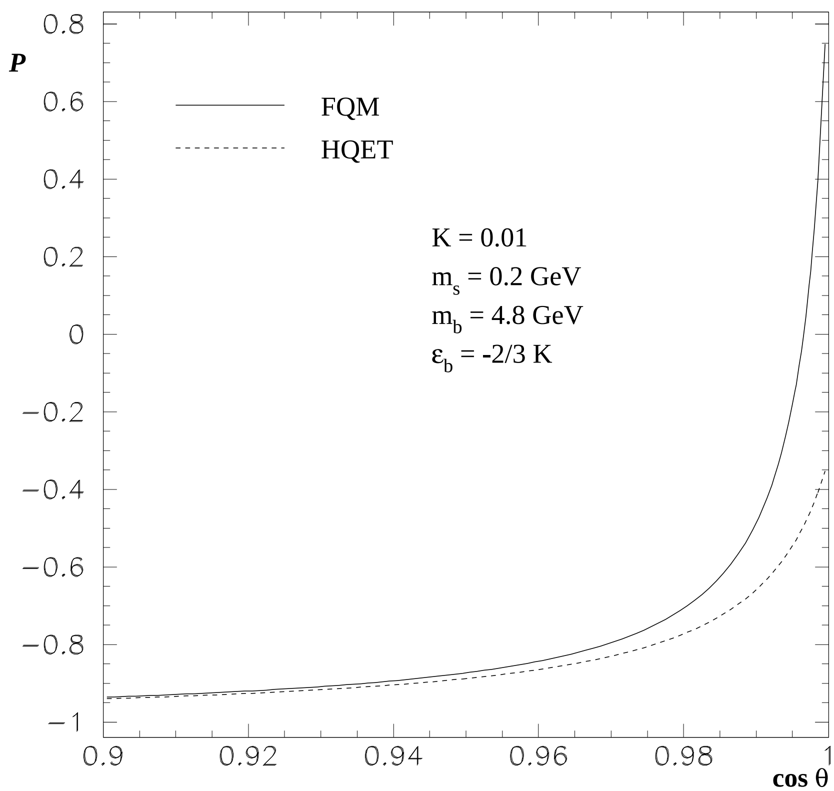
Figure 1 The fractional intensity I of photons in the forward cone $\cos\theta_0 \leq \cos\theta \leq 1$.

Figure 2 The photon polarization P in the inclusive Λ_b decay as a function of the photon direction with $K = 0.01$ and $\epsilon_b = -\frac{2}{3}K$.



I
x 10^-2
0.25
0.2
0.15
0.1
0.05
0
0.9
0.92
0.94
0.96
0.98
1
cos θ0
FQM
HQET
K = 0.01
m_s = 0.2 GeV
m_b = 4.8 GeV
ε_b = -2/3 K

FIG. 1:



P
FQM
HQET
K = 0.01
m_s = 0.2 GeV
m_b = 4.8 GeV
ε_b = -2/3 K
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.9
0.92
0.94
0.96
0.98
1
cos θ

FIG. 2: